Finding initial basic feasible solution

There are several methods of finding initial basic solutions. The methods to be discussed here are —

1. North west corner method.
2. column - minima method.
3. matrix - minima method.
4. Row minima method.
5. Vogel's Approximation method (VAM).

(1) North west corner method :

Step-1 :- Allocate in the cell at the North-west corner i,e, (1,1) cell of the ~~the~~ tableau, the maximum amount allowable so that either the capacity of the first row is exhausted or the demand of the first column is satisfied.

thus $x_{11} = \min(a_1, b_1)$.

Step-2 :- (i) If $b_1 > a_1$ we cross out the 1st row and adjust the associated amounts of supply and demand by substracting the allocated amount. Then move down vertically to the 2nd row and allocate in (2,1) cell an amount x_{21} where $x_{21} = \min(a_2, b_1 - x_{11})$

ii) If $a_1 > b_1$ we cross out the 1st column and adjust the associated amounts of supply and

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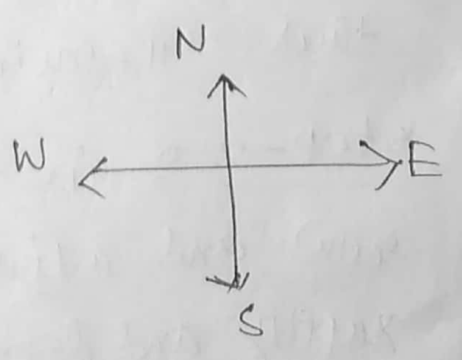
demand by substracting the allocated amount and then, we move right horizontally to the 2nd column and allocate in the (1,2) cell an amount x_{12} where $x_{12} = \min(a_1 - x_{11}, b_2)$

iii) If $a_1 = b_1$, there is a tie for the 2nd allocation. we cross out ~~cts~~ either 1st row or 1st column and give a zero supply (demand) in the uncrossed-out row (column) and then we make either $x_{12} = 0$ or $x_{21} = 0$.

Step-3 :- Now allocate at the next North-west corner i,e, at (2,2) cell, following step (1) and step 2 and continue these two steps untill all the rim requirements are satisfied. i,e, all availabilities are exhausted and all the demands are satisfied.

Example :- Find the basic feasible solution of the following transportation problem by North-West corner rule -

$$\begin{array}{c|ccccc|c} & \multicolumn{5}{c}{\text{Destinations}} & a_i \\ \hline \text{origins} & 2 & 11 & 10 & 3 & 7 & 4 \\ & 1 & 4 & 7 & 2 & 1 & 8 \\ & 3 & 9 & 4 & 8 & 12 & 9 \\ \hline b_j & 3 & 3 & 4 & 5 & 6 & \end{array}$$



Soln :-

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
b_j	3	3	4	5	6	

Since $\sum a_i = \sum b_j = 21$ so it is a balanced T.P

Allocation at (1,1) cell $x_{11} = \min(a_1, b_1)$

$= \min(4, 3) = 3.$

Since $(a_1 > b_1)$ we move horizontally to (1,2) cell

Allocation at (1,2) $x_{12} = \min(a_1 - x_{11}, b_2)$

$= \min(4-3, 3) = \min(1,3)$

$= 1$

Allocation at (2,2) cell (Since $a_1 < b_2$) is

$\min(a_2, b_2 - x_{12}) = \min(8, 3-1) = \min(8,2) = 2$

Allocation at (2,3) cell $x_{23} = \min(8-2, 4) = 4.$

Allocation at (2,4) cell $x_{24} = \min(a_2 - x_{22} - x_{23}, b_4)$

$= \min(2, 5) = 2$

Allocation at (3,4) cell $x_{34} = \min(a_3, b_4 - x_{24})$

$= \min(9, 5-2) = 3.$

allocation at (3,5) cell $= x_{35} = \min(a_3 - x_{34}, b_5)$

$= \min(6, 6) = 6.$

Now capacities are exhausted and all the demands are satisfied.

Now the final allocation are shown

	D1	D2	D3	D4	D5	ai
O1	(3) 2	(1) 11	10	3	7	4
O2	1	(2) 4	(4) 7	(2) 2	1	8
O3	3	9	4	(3) 8	(6) 12	9
bj	3	3	4	5	6	

thus the basic solution is

$x_{11}=3$; $x_{12}=1$; $x_{22}=2$; $x_{23}=4$; $x_{24}=2$;

$x_{34}=2$; $x_{35}=6$

The cost corresponding to this feasible solution

$= (3\times2)+(1\times11)+(2\times4)+(4\times7)+(2\times2)+(3\times8)+(6\times1$

$= 153.$

Example-2 :-

	D_1	D_2	D_3	D_4	a_i
O_1	4	6	9	5	16
O_2	2	6	4	1	12
O_3	5	7	2	9	15
b_j	12	14	9	8	

Soln.- Here min (16, 12) = 12. Therefore $x_{11}=12$ and allocate it in the cell (1,1). The demand D1 is satisfied and hence all other cells in the first column remain vacant. As $b_1=12<16=a_1$ therefore next allocation will be cell (1,2) and $x_{12}=$ min $(16-12, 14)=4$; Now the capacity of O_1 is exhausted. Next allocation will be in cell (2,2) and $x_{22}=$ min$(12, 14-4)=10$, proceeding similarly we get $x_{23}=2$; $x_{33}=7$ and $x_{34}=8$ and all the rim requirements are satisfied. The ~~solun~~ solution obtained in a B.F.S because the set of cells do not contain a loop

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(number of variables $4+3-1=6$; $(m+n-1)$ th^{m})

and the cost due to this assignment is

$4\times12 + 6\times4 + 6\times10 + 4\times2 + 2\times7 + 9\times8 = 226$ units.

H.W

Determine an initial B.F.S of the following problem by the method of North-west corner rule.

	D_1	D_2	D_3	D_4	a_i
O_1	2	5	4	7	4
O_2	6	1	2	5	6
O_3	4	5	2	4	8
b_j	3	7	6	2	18

Ans:- $x_{11}=3$; $x_{12}=1$

$x_{22}=6$; $x_{23}=0$

$x_{33}=6$; $x_{34}=2$.

Cost = 37 units.